TEL-RP2111 COMPUTERIZED CAVENDISH BALANCE

Declaration of Conformity

According to ISO/IEC Guide 22 and EN 45014.

We, ASONIK
declare under our sole responsibility that the product:
　　The Electronic Boards and Frame of the
　　Cavendish Balance Unit,
　　Type: USB
　　Input/Output: USB Interface

is in conformity with the following standard(s) or
other normative documents(s):

LVD - (EN 50366:2004(U), EN 50106:2000, EN60950-1:2001
　　　　EN 50371:2004, EN 60335-1:1999/2002/2004,
　　　　EN 61010-1:2001, EN 61187:2003)

EMC - (EN 55014-1:2004, EN 55014-2:1999, EN 55024:1998)

ROHS - (2002/95/EC, 2005/618/EC)

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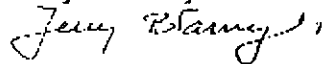
President of ASONIK
Jerzy Blaszczyk

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Introduction

The Computerized Cavendish Balance is designed to allow data to be taken with a computer, or with an optical lever arrangement. The period of the unit depends on the length of the tungsten wire and will vary from approximately 3½ to 4½ minutes. The electronics used to collect data are on the PCB board located inside the unit. They are designed such that the pendulous mode (to first order) is rejected. Thus the TEL-RP2111 Computerized Cavendish Balance is fairly immune to environmental vibrations. (This does not mean however, that you can bump the table when taking data or be reckless when moving the lead masses!)

The experiment can be completed using a dynamic data acquisition method in one (long) laboratory period. (This does not include replacing the tungsten wire if that is necessary.) Most of this time will be spent setting up and calibrating the unit. Setting up includes forcing the swinging masses to swing near the centerline of the Cavendish apparatus. Dampening the swing is required. If the tungsten wire needs to be replaced, we recommend that this task be performed before the laboratory begins.

The actual taking of data can be accomplished in a fairly short time by driving the unit in resonance with the boom oscillations and then allowing the boom to undergo free decay. A rough value for G can be obtained in as little as 3 oscillations, or about 15 minutes. Data can also be taken using the more traditional static method. After moving the large masses, the boom settles to its final value in about one hour.
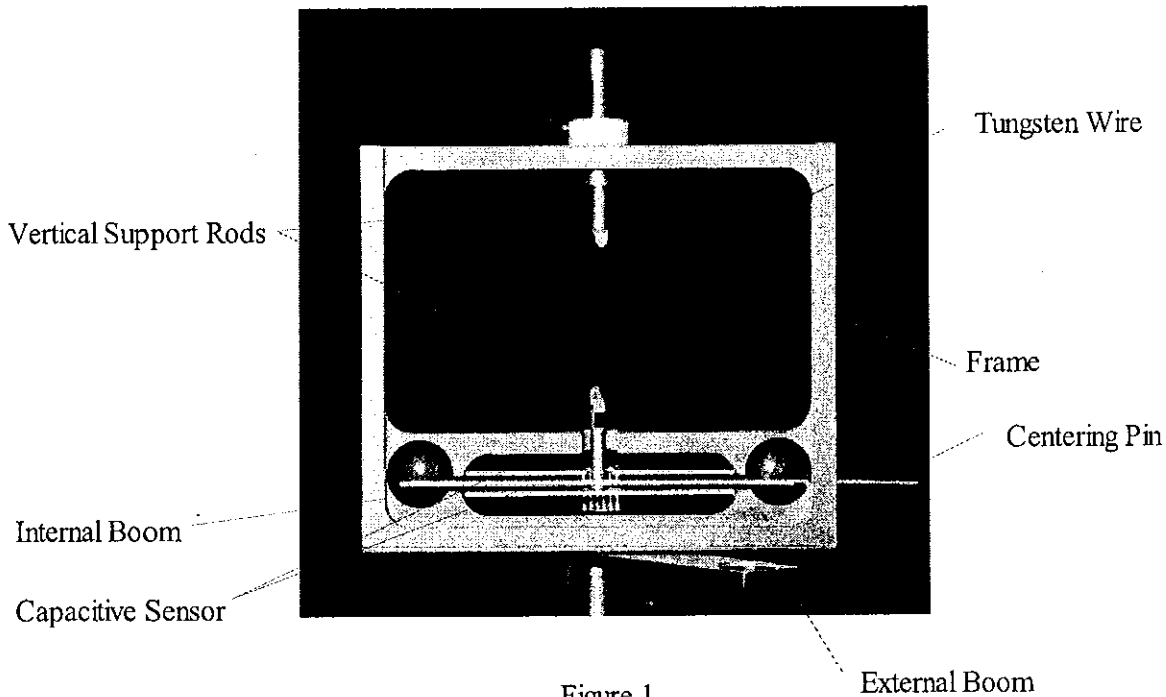

Figure 1

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Connection

Before the unit can be used, the accompanying software must be installed and the unit must be connected to the computer.

1. Insert the Cavendish Balance Software CD into your computer's CD-ROM drive.

2. Open windows explorer and navigate to your CD drive. Double click on "setup.exe" to start the install program. Follow the prompts until all software has been completely installed.

3. Connect the supplied USB cable to an available USB port on your computer and to the Cavendish Balance unit.

4. Start the Cavendish Balance program. If you have completed the installation correctly, you should see a value in milliradians in the upper left hand corner of the window.

For instructions on using the software, see Appendix 1, page 21.

Calibration

The unit can be calibrated using either of two methods. The simple method uses the uniform geometry of the unit to roughly calibrate the unit. The standard deviation for calibration with this method is about 5% of the full range the boom travels. The unit can also be calibrated using a laser to form an optical lever. This method can be quite accurate if proper care is taken.

SIMPLE CALIBRATION

1. Attach the boom to the solid rod. Attach the boom at the extreme end of the mounting hole. This allows the boom to be positioned as precisely as possible. See Figure 2 for detail.
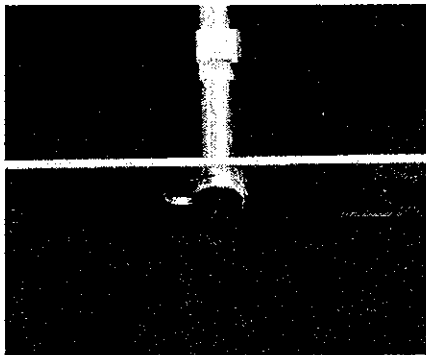

Figure 2

2. Install the boom and rod assembly into the balance. The boom should be centered vertically between the circuit boards, and should also be parallel to the circuit boards.

3. Insert both pieces of glass into the balance.

4. Connect the Cavendish balance to your computer.

5. In the Cavendish program, click on "Setup" and make sure that "Y maximum" is set to 70 mrad and "Y minimum" is set to -70 mrad. *Note: the value of +/- 70 mrad had been determined experimentally through repeated applications of this calibration method in conjunction with an optical lever.*

6. Carefully rotate the rod as far as it will go counterclockwise. Be sure that it does not spring back when you release it.

7. In the Cavendish program, in the setup window, click the "align left border" button. On the following dialog boxes, click yes and then yes again.

8. Carefully rotate the rod as far as it will go clockwise. Be sure that it does not spring back when you release it.

9. In the Cavendish program, in the setup window, click the "align right border" button. On the following dialog boxes, click yes and then yes again.

10. Click "OK" in the setup box. Your Cavendish balance is now ready to use.

OPTICAL LEVER CALIBRATION METHOD

1. Attach the boom to the solid rod. Attach the boom at the extreme end of the mounting hole.

2. Install the boom and rod assembly into the balance. The boom should be centered vertically between the circuit boards, and should also be parallel to the circuit boards.

3. Place the balance a distance L (at least 2 m) from a flat surface, such as a wall, as shown in figure 3. Arrange the laser so that it is at the same height as the mirror on the rod supporting the boom. Tape a piece of paper onto the wall so that you will be able to mark the position of the laser beam. This piece of paper should be approximately aligned such that the center of the paper is horizontally aligned with the balance unit.

4. Mark the initial position of the laser beam on the paper. Note and record the reading on the computer. Displace the boom by some small amount, ideally about 20mrad, and mark the laser's new position. Again, note and record the computerized reading. Find the distance between the two marks on your paper. This distance is S. Also calculate the difference between the computerized readings. Call this angle θ_{comp}.

5. If you have followed these instructions, we may approximate the wall as the edge of a circle. Hence, $\theta_{laser}=S/(2L)$ (The reflection of the laser introduces the 2)

6. If there is a concern that these approximations may not be valid (because the boom has been rotated through a large angle, or the mirror and the two laser spots do not form an approximately isosceles triangle) then θ can be calculated by using the law of cosines. If l_1 and l_2 are the distance from each laser spot to the mirror, then $\theta_{laser}=\arccos((l_1^2 + l_2^2 - S^2)/(2* l_1*l_2))/2$. (See figure 3)

7. When analyzing data, multiply all angle values by the calibration constant $\theta_{laser}/\theta_{comp}$. Alternatively, you may also multiply the "Y maximum" and "Y minimum" values by the same calibration constant and insert new value into the Setup menu. This ratio should be unchanged even when switching between a solid rod and tungsten wire.
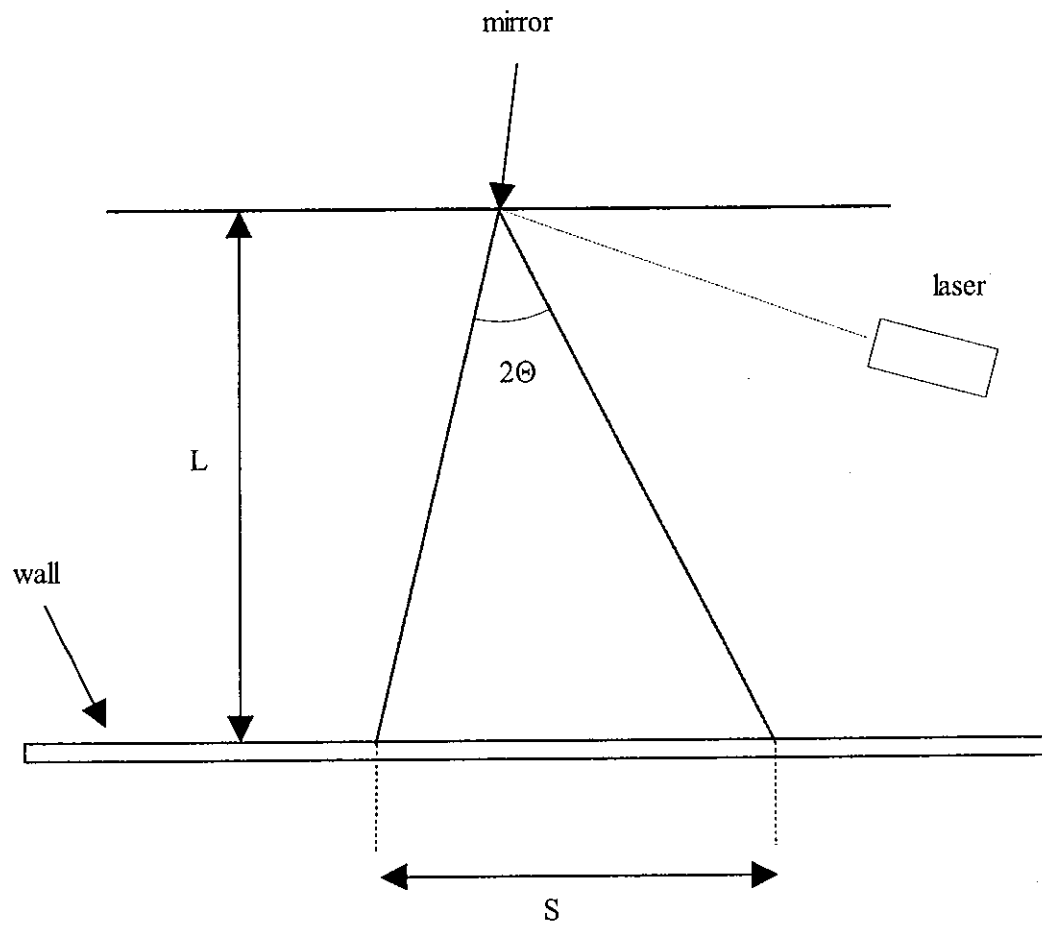
Figure 3

S - distance laser spot moves between turning points
L - distance of wall to mirror
θ - angle through which small boom moves
2θ - angle through which the reflected beam moves, (angle is 2θ because of angle doubling due to reflection)

REPLACING THE TUNGSTEN WIRE

The tungsten wire is only 25 microns in diameter and fairly fragile, therefore extreme care needs to be taken when tying this wire to the support rods (see figure 4). Carefully unroll 1 1/2 - 2 feet of wire. Tie the top wire first. Unscrew the screw from the top support just enough to wrap the wire around it. Take the free end of the wire and wrap around the screw. Be sure you wrap in the same direction the screw will be tightened. Make sure the other end of the wire is coming straight down from the top support before you tighten the screw. Now that the wire is in the right position, tighten the screw and that's it. Repeat for the bottom support rod. Once the bottom screw is tightened, cut the wire to be sure it doesn't contact and other parts of the balance.

It is imperative that you do not have any kinks in the wire.

This new design has made the process of tying the wire much less frustrating. However, it may be still be quicker and easier to find a young assistant with good hand eye coordination to tie the wire instead of tying it yourself.

Figure 4

CALCULATING G

The TEL RP2111 Computerized Cavendish Balance can be used to calculate G in two ways. The driven resonance method can be used to calculate G in about a half hour, while the more traditional static method can be completed in about 2 hours.

Driven Resonance Method

The driven resonance method of determining G has the advantage that the experimental data can be collected in a short time since one does not have to wait for the oscillations of the balance to damp away. Measurements can begin at any time the balance reaches a turning point. The large balls are rotated back and forth between the two extreme positions so that the force of gravity between the large balls and the boom is always doing positive work on the balance, and the amplitude builds up until the energy loss from damping is equal to the work done by the gravitational force. When turning the large spheres, it is important to stop them just before they contact the glass. Thus, determining G requires knowledge of the damping coefficient of the balance. This is most easily determined by measuring the amplitude decay as the balance is freely oscillating.

When freely oscillating, the angle of the boom as a function of time is given by

$$\theta(t) = \theta_e + A\, e^{-bt} \cos(\omega_1 t + \delta) \tag{1}$$

where

θ_e = equilibrium angle of the balance,

A = oscillation amplitude at $t = 0$,

b^{-1} = time for the amplitude to decay to $1/e$ of the initial value,

ω_1 = oscillation frequency; $\omega_1^2 = \omega_0^2 - b^2$, $\omega_1 = 2\pi/T$;

ω_0 = oscillation frequency in the absence of damping; $\omega_0^2 = K/I$,

T = oscillation period

K = torsion constant of the suspension fiber,

I = moment of inertia of the boom,

δ = phase of the oscillation at the time $t = 0$,

and where we have made the standard assumption that the damping torque is directly proportional to the angular velocity of the boom (We ignore damping due to internal friction in the tungsten wire. For more information, see Randal Peters' contributions in ch. 20 and 21 of CRC's **Vibration and Shock Handbook**, ed. Clarence deSilva, 2005). Figure 6 shows for a 50 minute time interval the measured displacement of the balance in free oscillation along with a least-squares fit to the function given in Equation 1.

Since the large masses are rotated at turning points of the oscillation, it is convenient to define the zero of time to occur at a turning point. In this case, the phase δ is specified by the requirement $d\theta/dt=0$ at $t=0$, and Equation 1 can be rewritten as

$$\theta(t) = \theta_e + A\,e^{-bt}\,[\cos(\omega_1 t) + b/\omega_1\,\sin(\omega_1 t)]. \qquad (2)$$

In what follows, we will concentrate on the turning points of the motion.

Figure 5: Cavendish Balance Free Decay
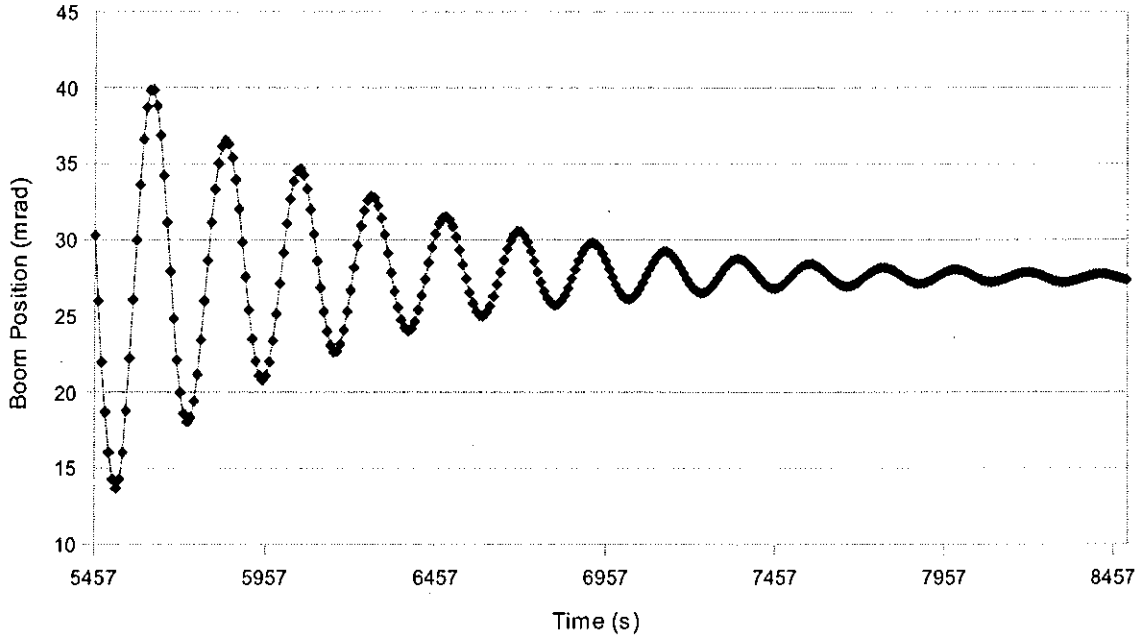
Let t_n be the time of the nth turning point ($t_n=(n-1)T/2$, the first turning point occurs at t=0), and let θ_n be the boom angle at the nth turning point, $\theta_n = \theta(t_n)$. The initial amplitude A is then just $\theta_1-\theta_e$. From Equation 2 we find

$$\theta_n = \theta_e + (\theta_1-\theta_e)(e^{-(n-1)bT/2})\,(-1)^{n-1} \qquad (3)$$

since $\omega_1 t_n = (n-1)\pi$. The factor $e^{-bT/2}$ occurs so often in the formulas below that it is convenient to define a separate symbol for it; let's call it x ($x \equiv e^{-bT/2}$). With this definition, Equation 3 becomes

$$\theta_n - \theta_e = (-x)^{n-1}\,(\theta_1 - \theta_e) \qquad (4)$$

which can also be written in the form:

$$(\theta_{n+1} - \theta_e) = -x\,(\theta_n - \theta_e) \qquad (5)$$

In free decay, x can be measured using any two adjacent turning points:

$$x = -(\theta_{n+1} - \theta_e)/(\theta_n - \theta_e). \qquad (6)$$

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One drawback to using Equation 6 to measure x is that it requires knowledge of the equilibrium angle, θ_e. By using three adjacent turning points, only differences in the turning point angles need be measured. Using Equation 5 twice, we find

$$x = -(\theta_{n+2} - \theta_{n+1})/(\theta_{n+1} - \theta_n). \tag{7}$$

Equation 7 is a very useful method to determine x. To reduce the measurement error on x, more turning points can be measured. If an odd number N of adjacent turning points are measured, multiple use of Equation 5 gives

$$x = 1 - (\theta_1 - \theta_N)/(\theta_1 - \theta_2 + \theta_3 - \theta_4 + ... - \theta_{N-1}). \tag{8}$$

NOTE: For mechanical oscillators in free decay, the positive and negative turning points can correspond to measurably different decay constants. Therefore, to improve accuracy in general, it is recommended that the results from Eq. 8 be averaged with the following:

$$x' = 1 - (\theta_2 - \theta_{N-1})/(\theta_2 - \theta_3 + \theta_4 - \theta_5 + ... - \theta_{N-2}) \tag{8a}$$

If the measurement error on each turning point is $\delta\theta$, the error on x can be shown to be

$$\delta x = \delta\theta \, (1-x)[(N-1)(1-x)^2 + 2x]^{1/2}/|\theta_1 - \theta_N|. \tag{9}$$

which has a broad minimum beginning around N=11 for x values typical of the balance.

Now let's consider the balance response to a resonant square-wave drive. We shall assume the gravitational torque exerted by the large masses on the boom when they are rotated from the center to the extreme positions is a constant, i.e. it is not appreciably changed by the small movements of the boom. With this assumption, the effect of the large masses is just to change the equilibrium angle of the boom. Before the drive is applied, the balance will either be at rest at the equilibrium angle θ_e, or else will be freely oscillating.

Figure 6: Driven Cavendish Balance
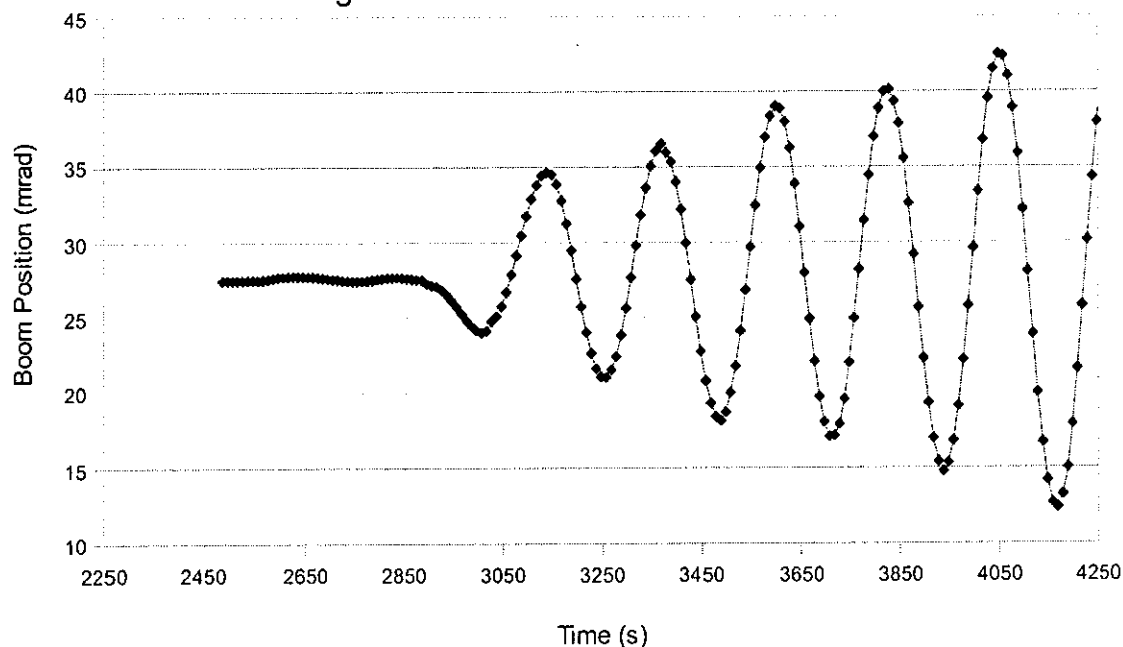
Let $\pm\theta_D$ be the change in the equilibrium angle when the large masses are rotated from the center position to either of the (symmetrically located) extreme positions. Suppose at time t=0 (a turning point if the balance is oscillating) the large masses are rotated to the extreme position where the new equilibrium angle is θ_e-θ_D. Then from Equation 2, the time dependence of the boom angle is

$$\theta(t) = (\theta_e\text{-}\theta_D) + (\theta_1\text{-}(\theta_e\text{-}\theta_D))\ e^{\text{-}bt}\ [\cos(\omega_1 t)\text{+}b/\omega_1\ \sin(\omega_1 t)] \qquad (10)$$

where θ_1 is the angle of the boom at t=0 (the first turning point). The boom angle at the second turning point is

$$\theta_2 = (\theta_e\text{-}\theta_D)\ \text{-x}\ (\theta_1\text{-}(\theta_e\text{-}\theta_D)). \qquad (11)$$

At the second turning point (t=T/2) the large masses are quickly rotated so that the new balance equilibrium angle becomes θ_e+θ_D. Thus, the boom angle at the third turning point (t=T) is

$$\theta_3 = (\theta_e\text{+}\theta_D)\ \text{-x}\ (\theta_2\text{-}(\theta_e\text{+}\theta_D)). \qquad (12)$$

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Each pair of adjacent turning points can be used to measure θ_D from which the gravitational constant G can be determined. It can be seen from Equations 11 and 12 that the solution for θ_D in terms of the two turning points θ_n and θ_{n+1} is

$$\theta_D = (-1)^n[(\theta_{n+1} - \theta_e) + x(\theta_n - \theta_e)]/(1+x). \tag{13}$$

The equilibrium angle θ_e can be eliminated from the measurement process if the results of two measurements of θ_D using three adjacent turning points are averaged:

$$\theta_D = (-1)^n[x\,\theta_n + (1-x)\,\theta_{n+1} - \theta_{n+2}]/[2(1+x)]. \tag{14}$$

To reduce errors, the results of an odd number N of adjacent turning points can be averaged:

$$\theta_D = [(1-x)(\theta_1 - \theta_2 + \theta_3 - \theta_4 + ... \theta_N) - \theta_1 + x\,\theta_N]/[(N-1)(1+x)] \tag{15}$$

where the error on θ_D is has contributions from the measurement error on the individual turning points $(\delta\theta_{D_\theta})$ and the error on x $(\delta\theta_{D_x})$:

$$\delta\theta_D = [\,\delta\theta_{D_\theta}{}^2 + \delta\theta_{D_x}{}^2\,]^{1/2} \tag{16a}$$

$$\delta\theta_{D_\theta} = \delta\theta\,[(N-1)(1-x)^2 + 2x]^{1/2}/[(N-1)(1+x)] \tag{16b}$$

$$\delta\theta_{D_x} = \delta x\,[2(\theta_1 - \theta_2 + \theta_3 - ... - \theta_{N-1}) + (\theta_N - \theta_1)]/[(N-1)(1+x)^2] \tag{16c}$$

assuming the measurements errors on the individual turning points are uncorrelated and equal to $\delta\theta$.

The minimum time required to collect the data needed to determine G is very short. Using Equation 7 to determine x and Equation 14 to determine θ_D, only 3 adjacent turning points when the balance is being resonantly driven and 3 adjacent turning points when the balance is in free decay need to be measured. Each set of 3 measurements require the balance to oscillate through one complete cycle which, depending on the length of the tungsten fiber, is about 4 minutes or less. If more accuracy is desired, any odd number N of adjacent data points can be used to determine x and θ_D using Equations 8 and 14, although 11 adjacent data points is optimum. (See equation 9, page 12)

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Static Method

The static method is conceptually much simpler than the driven resonance method. θ_D can be measured directly. Simply swing the large lead masses as far as they will go in one direction. Once the boom has settled down (about an hour), turn the large lead masses as far as they will go in the other direction. Once the boom has settled again, you can simply read off the two equilibrium angles. The difference between theses angles is equal to $2\theta_D$.

Determining G

Once θ_D has been measured through either the static or dynamic method, the gravitational constant G can be determined.

The torque exerted by the gravitational force of the large balls on the boom is balanced by a restoring torque of the tungsten fiber when it is rotated by the angle θ_D. The torque can be easily calculated if the following assumptions are made:

i. The balance is symmetric about the axis of rotation: the separation between the large and small masses is the same for both arms and for both equilibrium positions, the two large masses are identical, and the two small masses are identical.

ii. The mass of the aluminum beam is small and can be ignored.

iii. The positions of the large masses in both extreme positions is such that the gravitational force of the large masses on the small spheres is at right angles to the boom and in the plane perpendicular to the torsion fiber.

With these assumptions the gravitational torque τ_n of the large spheres on the nearby spheres is

$$\tau_n = 2\,G\,M\,m\,d\,/R^2 \qquad (17)$$

where

M = mass of each large sphere,

m = mass of each small sphere,

d = distance from the rotation axis to the center of the small sphere,

R = distance between the centers of the large and small spheres.

Equating the gravitational torque to the restoring torque of the fiber ($K\theta_D$) gives

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$$G = K \, \theta_D \, R^2 \, / \, (2 \, M \, m \, d). \qquad (18)$$

The torsion constant K can be determined from the equation for the oscillation frequency:

$$K = (4\pi^2/T^2 + b^2) \, I. \qquad (19)$$

For this balance the b^2 term is very small and can be ignored. The moment of inertia is the sum of the moment of inertia of the two small spheres (I_s) plus the moment of inertia of the aluminum beam (I_b):

$$I_s = 2(m \, d^2 + 2/5 \, m r^2) \qquad I_b = m_b \, (l_b{}^2 + w_b{}^2)/12 \qquad (20)$$

where r = radius of the small sphere, and the aluminum beam is assumed to be a uniform rectangle rotated about its center with mass m_b, length l_b, and width w_b.

Corrections:

The various assumptions that have been made in the above analysis introduce corrections to G for the non-ideal actual balance. The largest of these is the correction for the gravitational attraction of the large masses to the aluminum beam. A correction is also needed to account for the gravitational attraction of the large masses to the distant small masses. These and other corrections are discussed below.

1) Correction for the large sphere attraction to the distant small sphere.

The large spheres exert torques on the small spheres nearest them (Eq. 17), but they also exert oppositely directed torques on the distant small spheres. Compared to the torque exerted on the nearby sphere, this torque is reduced in magnitude both because the distant small spheres are much further away, and the gravitational force is no longer perpendicular to the lever arm. Including both effects, the torques exerted by the large spheres on the distant small spheres is given by

2 spheres: $$\tau_d = - 2 \, G \, M \, m \, d \, / R^2 \, f_d \qquad (28)$$

where
$$f_d = f = R^3 / [R^2 + (2d)^2]^{3/2}. \qquad (29)$$

For the Cavendish balance, $f_d = 3.5\%$.

2) Determining R:

To determine the separation R between the centers of the large and small spheres, it is easiest to assume the boom has been oriented so that at equilibrium, it is parallel to the glass plates and located midway between them. Ignoring the small rotation of the boom caused by the gravitational torques exerted by the large spheres, when the large spheres are just touching the glass windows R is given by

$$R = W/2 + R_L \qquad (30)$$

where W is the separation of the two outer surfaces of the glass panes, and R_L is the radius of the large spheres. In practice, the diameters of the two large spheres, D_{L1} and D_{L2}, are most easily measured. Furthermore, due to machining tolerances in the balance frame and imperfections in the surfaces of the large spheres, a small gap may exist between the surface of the glass window and one large sphere when the other large sphere is making contact with the glass surface. To account for these small gaps (typically less than 1 mm), it is convenient to average them and add the average to the separation W between the two outer glass surfaces. If G1 and G2 are the sizes of the gaps when the large spheres are in the two extreme positions, then R can be calculated from

$$R = W/2 + (D_{L1} + D_{L2})/4 + (G1+G2)/2. \qquad (31)$$

Corrections for the assumption that at equilibrium the boom is oriented parallel to the glass surfaces and midway between them turn out to be small. Suppose that the actual position of a small sphere at the equilibrium position is displaced from the assumed central position by an amount δ in a direction perpendicular to the boom. In this case, when the large spheres are in one position the true R will be smaller than the assumed R by the amount δ, but in the other extreme position the true R will be larger than the assumed R by δ. Thus, the strength of the gravitational force between the large and small spheres will be different for the two orientations of the large spheres. If the method used to obtain G treats the two extreme positions symmetrically, then the average of the two gravitational forces is the important quantity, and this average is only weakly sensitive to δ:

$$[1/(R+\delta)2 + 1/(R-\delta)^2] /2 \approx 1/R^2 [1 + 3 (\delta/R)^2]. \qquad (32)$$

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For example, if δ happens to be 3 mm, the correction to the average gravitational force from Eq. 33 for the Cavendish balance ($R \approx 4.6$ cm) is only 1.3%. When aligning the balance, the equilibrium position of the boom should be adjusted to within this accuracy. Note that the above argument assumes the two extreme positions of the large spheres are treated symmetrically. For the driven oscillation method, this condition will be better satisfied as more oscillations are averaged.

Corrections for the assumption that R (Eq. 32) doesn't change during the measurement depends on the method used to determine G. For the driven oscillation method averaged over many cycles, for each position of the large spheres the boom moves in an approximately symmetric manner with respect to the equilibrium position spending about equal amounts of time with R both larger and smaller than the equilibrium value. Using an argument similar to the one given above, the correction to G due to the assumption that R doesn't change is small. This is not true for the static method where G is determined from the change in the boom angle when the spheres are moved from one extreme position to the other. For this method, a correction to G from this effect of about 2% is required.

3) Correction for the gravitational torque exerted on the beam.

The aluminum beam can be approximated by a rectangle containing two holes that are used to support the small spheres. The large spheres exert a torque on the beam but less effectively than on the small spheres since much of the beam is further away from the center of the large spheres, and the gravitational force is not at right angles to the lever arm about the rotation axis. To estimate this torque, we will approximate the beam by a thin rod of length l_b having negligible thickness and width. If Δm is the mass of a small piece of the rod located a distance x from the axis of rotation, the gravitational torque exerted by one large sphere on the small piece is given by

$$\Delta \tau_b = G M \Delta m \, d / R^2 \, f \tag{21}$$

where f is a factor which accounts for the reduction of the torque compared to the case where the mass Δm was located at the center of the nearby small sphere:

$$f = x / d * [1 + ((d - x)/R)^2]^{-3/2}. \tag{22}$$

For regions near the position of the small sphere (x = d) f is approximately 1, but it dies away rapidly as x moves away from the small sphere position so that for x = 1/2 d, f = 0.26, and for x = -1/2 d, f = -0.036. The average of f over the area of the holes for the small spheres is within

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2% of 1. Thus, if we assume f=1 for the area of the holes, we will be making at most a 2% error on an already small correction. In that case, the affect of the holes can be accounted for by representing the beam by a uniform rod plus a point mass located at the center of the each hole whose mass m_h is equal to the mass of the aluminum removed to make the hole, but negative. In other words, the value of the mass of the hole just subtracts from the mass of the small sphere that sits in the hole. The value of m_h can be calculated from the density of aluminum ($\rho = 2.70$ g/cm^3) and the dimensions of the hole ($m_h = \rho\ V = 0.34$ g). Since the mass of the small spheres is about 14.6 g, this represents a 2% adjustment to the mass of the small sphere.

The torque exerted by a large sphere on the (now holeless) aluminum beam can be calculated by integrating $\Delta\tau_b$ (Eq. 22) over the length of the beam l_b:

1 sphere:
$$\tau_b = G\ M\ m_b\ d\ /\ R^2\ f_b \qquad (23)$$

where
$$f_b = \frac{R}{d}\frac{R}{l_b}\left[\frac{1}{\sqrt{1+\left(\frac{x-d}{R}\right)^2}}\left(\frac{d(x-d)}{R^2} - 1\right)\right]_{x_L}^{x_H} \qquad (24)$$

and where x_L and x_H are the positions of the two ends of the beam relative to the axis of rotation. Assuming the beam is symmetrically placed so that $x_L = -l_b/2$ and $x_H = +l_b/2$, Eq. 25 becomes

$$f_b = \frac{1}{d'}\frac{1}{2\ l'}\left[\frac{1+d'(l'+d')}{\sqrt{1+(l'+d')^2}} - \frac{1-d'(l'-d')}{\sqrt{1+(l'-d')^2}}\right] \qquad (25)$$

where $d' \equiv d/R$ and $l' \equiv l_b/(2R)$. For the Cavendish balance, $l' = 1.57$, $d' = 1.44$, and $f_b = 0.19$. Each large sphere exerts a torque on the beam, so the net torque exerted on the beam including the effects of the holes is

2 spheres:
$$\tau_b = 2\ G\ M\ (m_b\ f_b - m_h)\ d\ /\ R^2 . \qquad (26)$$

The ratio of the torques exerted by the large spheres on the beam over that exerted by the large spheres on the small masses is

$$\tau_b\ /\ \tau_n = (m_b\ f_b - m_h)\ /\ m \qquad (27)$$

which is about 7%.

Combining the results discussed above yields the following equation for the total torque exerted by the large spheres on the boom:

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$$\tau = \tau_n + \tau_d + \tau_b = 2 G M [(m - m_h)(1 - f_d) + m_b f_b] d / R^2. \tag{33}$$

Equating the total gravitational torque to the restoring torque of the fiber $(K\theta_D)$ gives

$$G = K \theta_D R^2 / (2 M [(m - m_h)(1 - f_d) + m_b f_b] d. \tag{34}$$

This equation incorporates analysis of all errors mentioned thus far, and therefore is the best equation to use to calculate G.

Appendix 1:The Computerized Cavendish Balance Software

INFORMATION BAR

Located at the top of the program window, the information bar contains three boxes. The **VALUE** box monitors the output from the Cavendish unit. The **ELASPED TIME** box notes the total time the experiment has run and the **DATA** box gives a read out of the data at the point where the cursor is placed. *This box is only active if the cursor is on the graph window.*

VIEW RECORD BAR

The Cavendish program will record data in either a single pass or a continuous mode. The **VIEW RECORD** bar describes the experimental set-up parameters. This bar will advise if the data on the screen is "live" or if it is data loaded from a previous experiment. The frequency at which data was taken, and the number of data points taken are also shown

COMMENT BAR

The **COMMENT** bar is located immediately below the **VIEW RECORD** bar and any comments about the experiment can be recorded here.

TOOL BAR

The Tool Bar just below the graph window allows the selection of the recording parameters. Data can be taken at 5 different rates selectable in the **SAMPLING RATE** box. The **NUMBER OF POINTS** box provides 7 "data widths" varying from 512 points to 32768 points and up to 19+ hours of recording time. The **RANGE** box has 6 different bipolar ranges from which selections can be made. The **SINGLE PASS** option, if checked, stops the data collection after a single pass. If that option is not checked then data is collected in the continuous mode.

Just below the tool bar on the left of the screen are eight buttons. The extreme upper left button is the **EXAMINE DATA** button. When this button is selected, a time and angle will be visible in the **INFORMATION BAR**.

Both the X and Y-axis can be "zoomed" by selecting the appropriate button, located to the right of the **EXAMINE DATA** button. After selecting the X or Y axis to zoom place the cursor in the active screen area and left click the mouse. To "unzoom" choose the appropriate box and left click the mouse and the screen will return to show the original experimental data. *Note: the maximum zoom is 512 points in time by approximately 1.5 milliradians.*

The **CONNECT DATA** button on the lower left is a toggle that shows the data as a collection of data points or shows the individual data points connected.

At the far right of the toolbar are the **SELECT POINTS** and **APPLY CORRECTION** buttons. In the top row, the **SELECT POINTS** button allows you to select one, two, or three points on the graph to define an offset, linear function, or quadratic curve. These can then be used to correct any drift that may become evident on long runs. We have empirically determined that changes in humidity are the main cause of drift. To select points first press the **SELECT POINTS** button. Then click on the graph window and select P1 from the pop up menu. Now select the first point to use for your drift correction. Repeat this process twice more to select points P2 and P3, if you want a higher-order correction.

Simply press the **APPLY CORRECTION** button to see the experimental data corrected with the offset, linear function, or quadratic curve. If you are not satisfied with the correction, there are two methods to adjust the correction. The first involves selecting new points to use for the correction. Deselect **APPLY CORRECTION** by clicking it again. Now, click on the graph window to select a new point for P1, P2, or P3. Click **APPLY CORRECTION** to see how the correction has changed. The second method of adjusting the correction involves adjusting the function coefficients directly. Click on the graph window, and select "set coef." from the pop up menu. You may now type in new values for A, B, and C. Click "OK" and then on the graph window again. Now click on "Manual Coefficients OFF" to toggle manual Coefficients on and see the data corrected using the values you entered. You may now continue to change the coefficients until you are satisfied with the fit.

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LOW PASS FILTER

The **LPF TIME CONSTANT** box allows the user to select 12 different Low Pass Filters's or to select no Low Pass Filter at all. This mode should seldom if ever have to be used. Start out all experiments with the *NO LPF* option. The common mode rejection of this unit is exceptional. For example if the small boom moves significantly in any motion other than a rotational movement the common mode rejection should prevent most all motion except the rotational motion to be recorded. If you find that this is the not case then the LPF may be of help.

RESET BUTTON

After completing a run, experimental parameters can be changed by selecting different sampling rates, number of data points, modes etc. Clicking on the **RESET** button will cause these changes to become effective.

RECORD BUTTON

To start taking data press **RECORD.** Any time a run is stopped, either manually or automatically, a warning box will pop up asking if you want to save the data before continuing.

SAVE/LOAD BUTTONS

Data can be saved by clicking on the **SAVE** button. The default format for saving data is *.cav. Two other formats (*.txt and *.xls) can be chosen. The latter two formats are convenient for exporting data to a spreadsheet for further analysis. The **LOAD** button allows one to load previously taken data. The program also creates a *temp.cav* file after about 2 minutes of data taking. Thereafter data is periodically saved in this temporary file. This is of use if, for example, the power were to go off while you were taking data over a long period of time. If that were to happen then you can recover the data up to the time in which the computer "crashed". This file is overwritten each time a new data run begins.

PRINT BUTTON

Data can be printed by selecting the **PRINT** button. The graph on the screen will be printed. In addition, the sample rate, number of points and file name will also be noted on the printed graph. The comments made in the **COMMENT** bar will also be printed if that option is chosen.

SET-UP BUTTON

This button allows you to set the min/max boundaries of the Cavendish unit.

"Y minimum" and "Y maximum" adjust the full scale range of the unit. By adjusting these (as discussed under **Calibration** on page 6) the balance output can me made to match the angle as measured with an optical lever.

The buttons "Adjust left border" and "Adjust right border" will each adjust the scale of the readout to make the current boom position correspond with the top (adjust right border) or bottom (adjust left border) edge of the screen.

The boom has been designed with tabs on the ends so that it is always rotating in a linear region of the capacitor plates. The total swing possible for the boom is 150 milliradians. The program however, cuts off any reading above +60 or below -60 millradians. Since the boom will only be swinging a few milliradians when performing the experiment, this cut off should not affect results.

Diagram 6 shows the dimensions of the Cavendish unit. From these dimensions, it is easy to calculate the maximum angle the boom can rotate. $\theta = ((30-18.7) / 2) / (150 / 2) = .075$ rad

18.7 mm   30 mm

150 mm
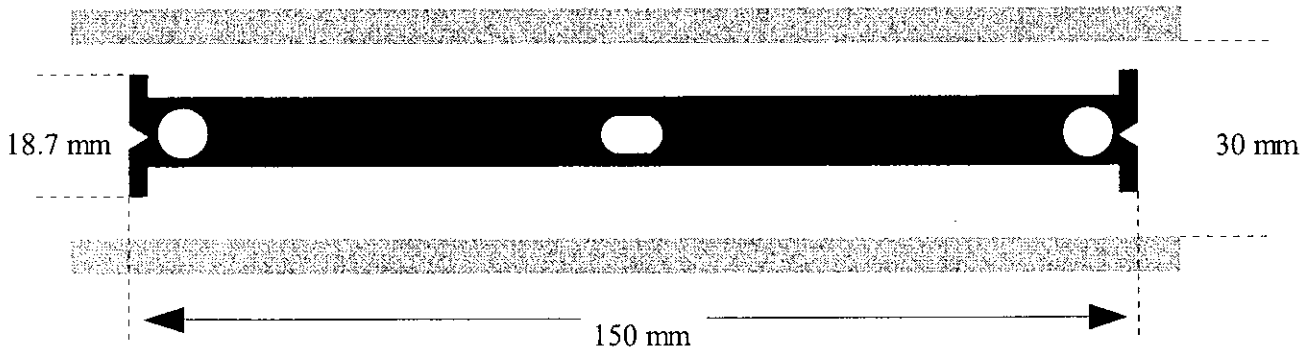
ZERO SETTING

The button just to the right of "RESET" sets the zero point of the boom. The Zero Point of the boom's swing will be *at the point* where the button was pressed. To set the zero point place the boom in the center of the plates. You can use the centering pin to temporarily hold the boom in place. Press the zero point button. The geometric center of the plates has now been established and the output will be read as milliradians on either side of this zero point.

EXIT BUTTON

The program will be closed if the **EXIT** button is selected.